COMMENTS AND ADDENDA

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Lorenz Numbers from Electron-Electron Scattering via a Screened Coulomb Interaction

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The electron-electron scattering Lorenz number L_e in the case of spin-independent scattering is computed using a simple model of two spherical bands. It is assumed that the s-d interaction is screened by the d electrons. The spatial dependence of the interaction is included via the form factor associated with the Wannier function of the d band. The form factor is included not only in the expression for the scattering rate, but also in the calculation of the Thomas-Fermi screening length.

The effect, on the Lorenz number L_e of metals, of electron-electron scattering via a screened Coulomb interaction can be readily computed from the formalism presented in a recent paper.[1] In that paper, which we shall refer to as I, the magnitude of the effect of the spin-independent, or Baber, scattering on the Lorenz numbers of transition metals was estimated, and specific applications were made to palladium and rhenium. It was found that when the spatial extent of the s-d interaction was ignored the calculated value of the Baber-scattering Lorenz number was the same value in all transition metals, 1.02×10^{-8} V^2K^{-2}. It was smaller than 1.02×10^{-8} V^2K^{-2} to an extent that depended on the metal, when the spatial dependence of the s-d interaction was included by introducing the form factor of the Wannier functions of the d band into the expression for the scattering rate. The calculation assumed that the s-d interaction was screened by the d electrons, but ignored their spatial extent in computing the Thomas-Fermi screening length κ. It is the purpose of this paper to point out that the calculation of the spatial dependence should include[2] a form factor in the expression for the Thomas-Fermi screening length as well as the form factor in the s-d interaction, and that this inclusion results in a Lorenz number which can be larger than 1.02×10^{-8} V^2 K^{-2}. Although this modification alters the numerical results, it does not change the essential feature of the model of Baber scattering which was employed in I.

We note that the matrix element for a screened Coulomb interaction is proportional to $(q^2+\kappa^2)^{-1}$, where κ is the Thomas-Fermi screening length and q is the change in momentum of the scattered electron. Now κ is given by $4\pi e^2N(0)|F(q)|^2$, where $N(0)$ is the total density of states at the Fermi level and $F(q)$ is the form factor defined in I. Numerical estimates with the specific form factors used in I for Pd and Re show, with this modified κ, that $\kappa \gg q$ over the entire range of q. Thus the arguments of I still obtain, and the integrals $\beta(m)$ in the expression for the Lorenz number L_e become

$$\beta^{(B)}(m) = \int_0^{2Q} d\eta\, \eta^{m-1}/\left|F(k_s\eta)\right|^2 \qquad (1)$$

instead of the expression presented as Eq. (4.13) of I.

Numerical calculations using Eq. (1), the form factor described in I, and the expression for L_e given as Eq.(2.11) of I, yield a Lorenz number of 1.27×10^{-8} V^2K^{-2} for Baber scattering in pure Pd. As in I, the Lorenz numbers of Pd-Ni calculated for electron-electron scattering can be modified by assuming that Baber and paramagnon scattering

are additive, and that the same amount of Baber scattering present in pure Pd is also present in the alloys. The modifications appropriate for the case of the local-enhancement model are shown in Fig. 1, where the results of including various percentages of Baber scattering are indicated. A similar modification to the uniform-enhancement model is not shown, for it results in Lorenz numbers which fall off far too fast with increasing nickel concentration to represent the data in a reasonable fashion. Note in Fig. 1 that it is possible either to fit the data at one point (pure Pd), which gives 87$\frac{1}{2}$% Baber scattering, or to project a Lorenz number which is truly independent of nickel concentration, which yields 75% Baber scattering. However, in view of the simplicity of the model, it is felt that such fits do not give a realistic estimate of the relative amounts of Baber and paramagnon scattering present in this system.

In the case of rhenium, the modification described above yields a Lorenz number $L_e = 1.108 \times 10^{-8} V^2 K^{-2}$. Although this is some 20% higher than the estimate presented in I, it still compares very favorably to the experimental value[3] of $0.9 \times 10^{-8} V^2 K^{-2}$.

We should like to thank Dr. M. J. Rice for suggesting the modification to our original calculation. We are indebted to Dr. James R. Cullen for valuable discussions. We should also like to thank D. J. Gillespie for his assistance in programming the calculations.

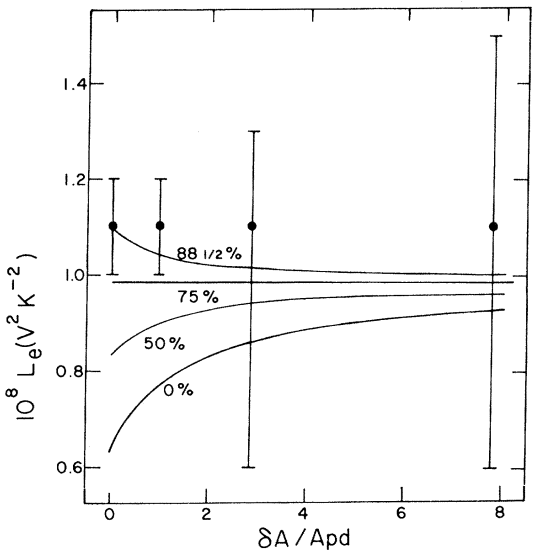

FIG. 1. Comparison of calculated and measured values of L_e for Pd and dilute *Pd*-Ni alloys. A is the coefficient of the T^2 term in the electrical resistivity, and $\delta A = A_{\text{alloy}} - A_{\text{Pd}}$. Solid dots and error bars represent values of L_e extracted from the data. The 0% curve is the result of a calculation with the local-enhancement model of paramagnon scattering with $\bar{I}_{\text{Pd}} = 0.9$, $k_s = 0.9125 \times 10^8$ cm^{-1}, and $k_d = 2k_s$. The other curves are the results of modifying the calculation by adding a concentration-independent amount of Baber scattering to the paramagnon scattering. The percentages indicate for each curve the amount of the total resistivity in pure Pd which is assumed to be Baber scattering.

[1]J. T. Schriempf, A. I. Schindler, and D. L. Mills, Phys. Rev. 187, 959 (1969).

[2]M. J. Rice (private communication).

[3]J. T. Schriempf, Phys. Rev. Letters 20, 1034 (1968).

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Electron-Electron Scattering in the High-Temperature Thermal Resistivity of the Noble Metals

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Recently reported values for the Lorenz function of the noble metals show, at high temperatures, deviations from the expected standard value. These deviations can be satisfactorily explained, both in magnitude and temperature variation, in terms of normal electron-electron scattering.

The recently completed measurements of the high-temperature transport properties of the noble metals[1] show an interesting feature in that none of the electronic Lorenz functions L_e actually reach the standard Sommerfeld value $L_0 = \frac{1}{3}(\pi k/e)^2$ even at the highest temperatures investigated, contrary to theoretical expectations for electron-phonon scattering.[2] This feature is shown in Fig. 1, where L_e is plotted versus a reduced temperature T/Θ_R. L_e has been calculated by subtracting the phonon contribution to the thermal conductivity, given by White,[3] from the measured total conductivity; Θ_R is the Debye temperature obtained from electrical-resistivity data, and has been taken